HIGHER SECONDARY II YEAR

MATHEMATICS

Model Question Paper - 1

Time : 2.30 Hours | Marks : 90

Part - I

All questions are compulsory **$20 \times 1 = 20$**

Choose the correct answer

1. Let A be a square matrix all of whose entries are integers. Then which one of the following is true ?

   a) If det (A) = ± 1, then A^{-1} exists but all its entries are not necessarily integers

   b) If det (A) $\neq \pm 1$, then A^{-1} exists and all its entries are non integers

   c) If det (A) = ± 1, then A^{-1} exists and all its entries are integers

   d) If det (A) = ± 1, then A^{-1} need not exist

2. If $A = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$ then A^{12} is

   a) $\begin{bmatrix} 0 & 0 \\ 0 & 60 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 0 & 5^{12} \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ $|\vec{a}| = 7$, $|\vec{b}| = 5$, $|\vec{c}| = 3$ then angle between vectors $\vec{b}$ and $\vec{c}$ is

   a) 60^0 b) 30^0 c) 45^0 d) 90^0

4. If $\vec{a} \times (\bar{b} \times \bar{c}) + \vec{b} \times (\bar{c} \times \bar{a}) + \vec{c} \times (\bar{a} \times \bar{b}) = \vec{x} \times \vec{y}$ then

   a) $\bar{x} = 0$ b) $\bar{y} = 0$ c) $\bar{x}$ and $\bar{y}$ are parallel

   d) $\bar{x} = 0$ or $\bar{y} = 0$ or $\bar{x}$ and $\bar{y}$ are parallel

5. Let A and B denote the statements

   A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

   B : $\sin\alpha + \sin\beta + \sin\gamma = 0$. If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = \frac{-3}{2}$ then

a) A is true and B is false b) A is false and B is true

c) both A and B are true d) both A and B are false

6. The conjugate of $i^{13} + i^{14} + i^{15} + i^{16}$ is

a) 1 b) –1 c) 0 d) –i

7. The eccentricity of an ellipse with its centre at the origin is $\frac{1}{2}$. If one of the directrices is x = 4, then the equation of the ellipse is

a) $3x^2 + 4y^2 = 1$ b) $3x^2 + 4y^2 = 12$ c) $4x^2 + 3y^2 = 12$

d) $4x^2 + 3y^2 = 1$

8. One of the foci of the rectangular hyperbola xy = 18 is

a) (6, 6) b) (3, 3) c) (4, 4) d) (5, 5)

9. $\lim\limits_{x \to \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)$ is

a) e^4 b) e^2 c) e^3 d) 1

10. If $y = 6x - x^3$ and x increases at the rate of 5 units per second, the rate of change of slope when x = 3 is

a) – 90 units/sec b) 90 units/sec

c) 180 units/sec d) –180 units/sec

11. The Rolles' constant for the function $y = x^2$ on [–2, 2] is

a) $\frac{2\sqrt{3}}{3}$ b) 0 c) 2 d) –2

12. The point on the curve $x = at^2$, $y = 2at$, at which the tangent is at 45^0 to the x axis is

a) (2a, a) b) (a, –2a) c) $(2a, 2\sqrt{2}\,a)$ d) (a, 2a)

13. The area bounded by the parabola $x^2 = 4 - y$ and the lines y = 0 and y = 3 is

a) $\frac{14}{3}$ sq.units b) $\frac{28}{3}$ sq. units c) $4\sqrt{3}$ sq. units d) $\frac{56}{3}$ sq.units

14. The value of $\int_0^1 x\,(1-x)^4\,dx$ is

a) $\frac{1}{12}$ b) $\frac{1}{30}$ c) $\frac{1}{24}$ d) $\frac{1}{20}$

15. The degree of the differential equation $\frac{\left[\left(1+\frac{dy}{dx}\right)^3\right]^{2/3}}{\frac{d^3y}{dx^3}} = C$ where C is a constant is

a) 1 b) 3 c) –2 d) 2

16. The particular integral of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$

   a) $x\,e^{3x}$ b) e^{3x} c) $\frac{x^2}{2}e^{3x}$ d) ∞

17. The order of an element a of a group is 10. (ie) 0(a) = 10 Then the order of $(a^2)^{-1}$ is

   a) 10 b) 5 c) 2 d) 1

18. The value of $[3] +_{11} ([5] +_{11} [6])$ is

   a) [0] b) [1] c) [2] d) [3]

19. When two dice are thrown the probability of getting one five is

   a) $\frac{25}{36}$ b) $\frac{5}{36}$ c) $\frac{1}{36}$ d) $\frac{5}{18}$

20. If in a poission distribution P (X = 0) = K, then the variance is

   a) $\log\frac{1}{K}$ b) log K c) e^{λ} d) $\frac{1}{K}$

Part - II

Answer any Seven questions. Question 30 is Compulsory **7 × 2 = 14**

21. Consider the system of linear equation $x_1 + 2x_2 + x_3 = 3$, $2x_1 + 3x_2 + x_3 = 3$, $3x_1 + 5x_2 + 2x_3 = 1$ find the solution if exists.

22. For any vector $\vec{a}$, prove that the value of $\left(\vec{a}\times\vec{i}\right)^2 + \left(\vec{a}\times\vec{j}\right)^2 + \left(\vec{a}\times\vec{k}\right)^2 = 2a^2$.

23. If the cube roots of unity are 1, ω, ω^2 then find the roots of the equation $(x - 1)^3 + 8 = 0$.

24. Find the condition that y = mx + c may be a tangent to the conics parabola $y^2 = 4ax$.

25. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in [0, 1].

26. Find $\frac{\partial w}{\partial t}$, if $w = x^2y - 10y^3z^3 + 43x - 7\tan(4y)$ where $x = t$, $y = t^2$, $z = t^3$

27. Find the value of $\int_{-1}^{2} |x|\,dx$.

28. Solve $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}\,dy = 0$.

29. Prove that the set of all 4th roots of unity forms an abelian group under multiplication.

30. For the probability density function $f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \le 0 \end{cases}$ find F(2).

Part - III

Answer any Seven questions. Question No.40 is compulsory. **7 × 3 = 21**

31. Solve by matrix inversion method x + y = 3, 2x + 3y = 8

32. What is the radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is

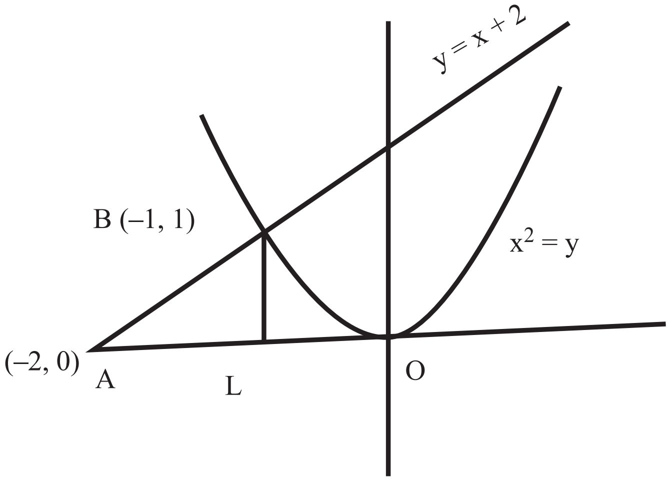
cut by the plane $x + 2y + 2z + 7 = 0$.

33. Find the real and imaginary parts of the complex number $Z = \frac{3i^{20} - i^{19}}{2i - 1}$

34. The tangent at any point of the rectangular hyperbola $xy = c^2$ makes intercepts a, b and the normal at the point makes intercepts p, q on the axes. Prove that $ap + bq = 0$.

35. Find the point of inflection to the curve $y = \sin^2 x$ where $\frac{-\pi}{2} < x < \frac{\pi}{2}$.

36. Compute the area of the figure enclosed by the curves $x^2 = y$, $y = x + 2$ and x axis.



37. Solve $x^2 dy + y(x + y)\,dx = 0$

38. Find the order of each element of the group $(z_{12}, +_{12})$.

39. In a binomial distribution the arithmetic mean and variance are respectively 4 and 3. If the random variable X denotes the number of successes in the corresponding experiment then find $P(x = 2) / P(x = 3)$.

40. Verify Euler's theorem for $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$

Part - IV

Answer all the questions **7 × 5 = 35**

41. a) Examine the consistency of the following system of equations. If it is consistent then solve using rank method.

$4x + 3y + 6z = 25$, $x + 5y + 7z = 13$, $2x + 9y + z = 1$

or

b) Find the vector and cartesian equations to the plane through the point (–1, 3, 2) and perpendicular to the plane $x + 2y + 2z = 5$ and $3x + y + 2z = 8$.

42. a) $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors of magnitudes a, b, c respectively. Also $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ = abc. Then prove that $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$.

or

b) If α and β are the roots of $x^2 - 2x + 2 = 0$ and $\cot\theta = y + 1$ show that

$$\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha-\beta} = \frac{\text{Sin } n\theta}{\text{Sin}^n \theta}$$

43. a) Find directrix, latus rectum of the ellipse $6x^2 + 9y^2 + 12x - 36y - 12 = 0$ also draw the diagram.

or

b) The path of a ship can be described by a hyperbolic model centered at the origin, relative to two stations on the shore 168 miles apart that are located at the foci. If the ship is 40 miles south of the centre of the hyperbola, find the equation of the hyperbola.

44. a) Find the values of x, y whose product xy = 64 and such that $4x + 27y^3$ is maximum.

or

b) Prove that the sum of the intercepts on the co-ordinate axes of any tangent to the curve $x = a\,\text{Cos}^4\theta$, $y = a\,\text{Sin}^4\theta$, $0 \le \theta \le \frac{\pi}{2}$ is equal to a.

45. a) $u = \tan^{-1}\left(\frac{x}{y}\right)$ Verify $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 u}{\partial y\,\partial x}$

or

b) The plane region bounded by the curve $y = \sqrt{\cos x}$, $0 \le \theta \le \frac{\pi}{2}$ and the lines x = 0, y = 0 is rotated about x axis. Find the volume of the solid.

46. a) Derive the formula for the volume of a right circular cone with radius 'r' and height 'h'. using integration.

or

b) A Bank pays interest by continuous compounding that is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accures at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year.

47. a) Show that the set of all matrices of the form $\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $a \in R - \{0\}$ is an abelian group under matrix multiplication.

or

b) Solve : $x\frac{dy}{dx} - y = (x-1)\,ex$

HIGHER SECONDARY II YEAR

MATHEMATICS

Model Question Paper - 2

Time : 2.30 Hours Marks : 90

Part - I

All questions are compulsory **$20 \times 1 = 20$**

Choose the correct answer

1. If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & K & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has an inverse, then the value of K

   a) K is any real number b) $K = -4$ c) $K \neq -4$ d) $K \neq 4$

2. The value of $\begin{vmatrix} \text{Cos } 15^0 & \text{Sin } 15^0 \\ \text{Cos } 45^0 & \text{Sin } 45^0 \end{vmatrix} \times \begin{vmatrix} \text{Cos } 45^0 & \text{Cos } 15^0 \\ \text{Sin } 45^0 & \text{Sin } 15^0 \end{vmatrix}$ is

   a) $\frac{1}{4}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{-\sqrt{3}}{4}$ d) $\frac{-1}{4}$

3. $\vec{r} = s\vec{i} + t\vec{j}$ is the equation of

   a) a straight line joining the points $\vec{i}$ and $\vec{j}$ b) xoy plane

   c) yoz plane d) zox plane

4. If $\vec{a} = \vec{i} + \vec{j} - \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + \vec{k}$ $\vec{c} = \vec{i} - \vec{j} - \vec{k}$ then the value of $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is

   a) $\vec{i} - \vec{j} + \vec{k}$ b) $2\vec{i} - 2\vec{j}$ c) $3\vec{i} - \vec{j} + \vec{k}$ d) $2\vec{i} + 2\vec{j} - \vec{k}$

5. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, The minimum value of $|z_1 - z_2|$ is

   a) 0 b) 2 c) 7 d) 17

6. If $-i + 3$ is a root of $x^2 - 6x + K = 0$ then the value of K is

   a) 5 b) $\sqrt{5}$ c) $\sqrt{10}$ d) 10

7. The distance between the foci of the ellipse $9x^2 + 5y^2 = 180$ is

   a) 4 b) 6 c) 8 d) 2

8. If the foci of an ellipse are (3, 0), (–3, 0) and the eccentricity is 1/2 then the equation of the ellipse is

a) $\frac{x^2}{16}+\frac{y^2}{9}=1$ b) $\frac{x^2}{9}+\frac{y^2}{4}=1$, c) $\frac{x^2}{36}+\frac{y^2}{27}=1$ d) $\frac{x^2}{25}+\frac{y^2}{9}=1$

9. A particle's velocity V at time t is given by $V = 2e^{2t}\cos\left(\pi t/3\right)$ what is the least value of t at which the acceleration becomes zero ?

a) 0 b) $\frac{3}{2}$ c) $\left(\frac{3}{\pi}\right)\tan^{-1}\left(\frac{6}{\pi}\right)$ d) $\frac{3}{\pi}\cot^{-1}\left(\frac{6}{\pi}\right)$

10. The 'c' of Lagrange's mean value theorem for the function $f(x) = x^2 + 2x - 1$, $a = 0$, $b = 1$ is

a) –1 b) 1 c) 0 d) 1/2

11. If $u = \log\left(\frac{x^2+y^2}{xy}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is

a) 0 b) u c) 2u d) u^{-1}

12. $\frac{\partial^2}{\partial x\,\partial y}(x^y) =$

a) $x^{y-1}(1 + y\log x)$ b) $y(y-1)x^{y-2}$

c) $x^{y-1} + (y-1)x^{y-2}$ d) $x^y(x - y\log x)$

13. The value of $\int_0^{\pi/2}\frac{\sin x - \cos x}{1+\sin x\cos x}$ is

1) $\pi/2$ 2) 0 3) $\pi/4$ 4) π

14. The plane region is enclosed by the line $x + y - 2 = 0$, x axis and y axis. The volume generated by this region when it is revolved about x - axis is

a) $\frac{\pi}{3}$ cu. unit b) $\frac{2\pi}{3}$ cu. units c) $\frac{4\pi}{3}$ cu. units

d) $\frac{8\pi}{3}$ cu. units

15. The solution of the equation $\frac{dy}{dx}=\frac{y}{x}+\tan\left(\frac{y}{x}\right)$ is

a) $cx = \tan^{-1}\left(\frac{y}{x}\right)$ b) $cy = \cos\left(\frac{x}{y}\right)$ c) $cx = \sin\left(\frac{y}{x}\right)$ d) $cy = \sin\left(\frac{x}{y}\right)$

16. The differential equation $\left(\frac{dx}{dy}\right)^2 + 5y^{1/3} = x$ is

a) of order 2 and degree 1 b) of order 1 and degree 2

c) of order 1 and degree 6 d) of order 1 and degree 3

17. If P is T and q is F, then which of the following have the truth value T ?

(i) $p \vee q$ (ii) $\sim p \vee q$ (iii) $p \vee \sim q$ (iv) $P \wedge \sim q$

a) (i), (ii), (iii) b) (i), (ii), (iv) c) (i), (iii), (iv) d) (ii), (iii), (iv)

18. The set of all nth roots of unity form an abelian group under multiplication. The inverse of the element $\cos (n-1)\frac{2\pi}{n} + i\sin (n-1)\frac{2\pi}{n}$ is

a) $\cos (n-1)\pi + i \sin (n-1)\pi$ b) $\cos n\pi + i \sin n\pi$ c) $\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$

d) $\cos 2\pi + i \operatorname{Sin} 2\pi$

19. The probability that any number between 1 and 20 be divisible either by 3 or by 7 is

a) $2/5$ b) $1/3$ c) $4/9$ d) $5/10$

20. If f(x) is a p.d.f of a normal variate X and $X \sim N(\mu, \sigma^2)$ then $\int_{-\infty}^{\mu} f(x)\,dx$

a) undefined b)1 c) 0.5 d) –0.5

Part - II

Answer any Seven questions Question 30 is Compulsory **7 × 2 = 14**

21. Find the inverse of A, where $A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$

22. For any two vectors $\vec{a}$ and $\vec{b}$ prove that $|\bar{a} + \bar{b}| \le |\bar{a}| + |\bar{b}|$

23. Compute the square roots of $Z = -1 - i$

24. Compute real and imaginary parts of $Z = \frac{i-4}{2i-3}$

25. Find the equation of the parabola, if the curve is open rightward, vertex is (2, 1) and passing through the point (6, 5).

26. Prove that the function $f(x) = \sin x + \cos 2x$ is not monotonic on the interval $\left[0, \frac{\pi}{4}\right]$

27. Estimate $\sqrt{4.001}$ by approximate value using differentials.

28. Find the value of $\int_0^{\pi/2} e^{ax} \cos bx\, dx$,.

29. Find the degree and order of the equation $x\frac{dy}{dx} = y + \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2}$.

30. Out of 13 applications for a job, there are 8 men and 5 women. It is decided to select 2 persons for the job. Find the probability that atleast one of the selected person will be a woman.

Part - III

Answer any SEVEN questions **7 × 3 = 21**

Question No.40 is compulsory.

31. Find the rank of the matrix.

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$$

32. A particle is acted upon by constant forces $4\vec{i}+\vec{j}-3\vec{k}$ and $3\vec{i}+\vec{j}-\vec{k}$ which displace it from a point $\vec{i}+2\vec{j}+3\vec{k}$ to the point $5\vec{i}+4\vec{j}+\vec{k}$. What is the work done in standard units by the force.

33. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$,then find (a, b).

34. Find the equation of a point which moves so that the sum of its distances from (–4, 0) and (4, 0) is 10.

35. Find the points of inflection of y = tan x in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

36. If u = log (tan x + tan y + tan z), prove that $\sum \text{Sin } 2x \frac{\partial u}{\partial x} = 2$.

37. Find the area between the curves $y = e^x$ and $y = e^{-x}$ from x = –1 to x = 2.

38. Use truth table to determine whether the statement $((\sim p) \vee q) \vee (p \wedge (\sim q))$ is a tautology.

39. Solve $2y \cot x \frac{dy}{dx} = 1 + y^2$

40. A random variable X has the probability mass function as follows :

X	–2	3	1
P (X = x)	$\frac{\lambda}{6}$	$\frac{\lambda}{4}$	$\frac{\lambda}{12}$

Then find E(x).

Part - IV

Answer all the questions **7 × 5 = 35**

41. (a) Investigate for what values of λ, μ the simultaneous equations x + y + z = 6, x + 2y + 3z = 10, x + 2y + λz = μ have (i) no solution (ii) a unique solutions (iii) an infinite number of solutions

or

(b) Altitudes of a triangle are concurrent prove by vector method.

42. (a) P represents the variable complex number z, Find locus of P if $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$

or

(b) Find the axes, centre, focus and directrix of the parabola $y^2 - 6y - 8x + 25 = 0$.

43. (a) The orbit of the earth around the sun is elliptical in shape with sun at a focus. The semi major axis is of length 92.9 million miles and eccentricity is 0.017. Find how close the earth gets to sun and the greatest possible distance between the earth and the sun.

(or)

(b) Show that the value of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$

44. (a) Evaluate $\lim_{x\to\frac{\pi}{2}^-}(\tan x)^{\cos x}$

or

(b) If $[\vec{b}, \vec{c}, \vec{d}] = 24$ and $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})+(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})+K\vec{a}=0$ then find the value of K/16.

45. (a) Trace the curve $y^2 = 2x^3$

or

(b) The curves y = sinx and y = tanx touch each other at x = 0. What is the area bounded by them and the line $x = \frac{\pi}{3}$?

46. (a) Find the length of the curve $\left(x/a\right)^{2/3}+\left(y/a\right)^{2/3}=1$

or

(b) Let $L(y)=\frac{d^2y}{dx^2}+7\frac{dy}{dx}+3y$. If $y_1(x)$ and $y_2(x)$ are solutions of $L(y)=\text{Sin}x$ and $L(y)=\text{Cos}x$ respectively then find the differential equation having $y_1(x) + y_2(x)$ as a solution.

47. (a) Show that $(z_n, +_n)$ forms group.

or

(b) The probability distribution of a discrete random variable x is given by

X	–2	2	5
P (x=x)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

then find $4E(x^2) - Var(2x)$.

HIGHER SECONDARY II YEAR

MATHEMATICS

Model Question Paper - 3

Time : 2.30 Hours Marks : 90

Part - I

All the questions are compulsory. **20 x 1 = 20**

Choose the correct answer :

1. The given system of equations $x + 2y + 3z = -1$, $2x + 5y - 6z = 3$, $8x + 20y - 24z = 12$ is

   a) consistent and has unique solution

   b) consistent and has infinite no. of solutions

   c) inconsistent and has no solution

   d) consistent and has trivial solution

2. Which of the following statement is false

   a) The rank of a null matrix is zero

   b) The rank of the unit matrix of order n is unity

   c) Rank of an $m \times n$ matrix cannot exceed the minimum of m and n

   d) Equivalent matrices have the same rank

3. Direction ratios of y and z axis are respectively

   a) (1, 0, 0), (0, 1, 0) b) (1, 0, 0), (0, 0, 1)

   c) (0, 0, 1), (0, 1, 0) d) (0, 1, 0), (0, 0, 1)

4. If one end of the chord passing through the centre of the sphere $(x-2)^2 + (y-1)^2 + (z+6)^2 = 18$ is $(3, 2, -2)$ then the other end of the chord is

   a) $\left(\frac{5}{2}, \frac{3}{2}, -4\right)$ b) $\left(\frac{-1}{2}, \frac{-1}{2}, -2\right)$ c) $(1, 0, -10)$ d) $(-1, 0, 10)$

5. The conjugate of $\overline{Z} + 3i$ is

   a) $Z - 3i$ b) $Z + 3i$ c) $-Z + 3i$ d) $-Z - 3i$

6. The real part of $e^{Z + 2\pi i}$ is

   a) $\cos y$ b) $\sin y$ c) $e^x \cos y$ d) $e^x \sin y$

7. If a parabolic reflector is 20cm in diameter and 5cm deep, the distance of the focus from the centre of the reflector is

   a) 10 cm b) 6 cm c) 5 cm d) 15 cm

8. The normal to the rectangular hyperbola xy = 9 at $\left(6, \frac{3}{2}\right)$ meets the curve again at

a) $\left(\frac{3}{8}, 24\right)$ b) $\left(-24, \frac{-3}{8}\right)$ c) $\left(\frac{-3}{8}, -24\right)$ d) $\left(24, \frac{3}{8}\right)$

9. The curve $y = ax^3 + bx^2 + cx + d$ has a point of inflection at x = 1 then

a) a + b = 0 b) a + 3b = 0 c) 3a + b = 0 d) 3a + b = 1

10. The characteristic function of irrational numbers is

a) everywhere differentiable and everywhere continuous

b) everywhere differentiable and everywhere discontinuous

c) nowhere differentiable and everywhere continuous

d) nowhere differentiable and everywhere discontinuous

11. The asymptote to the curve $y^2 (1 + x) = x^2 (1 - x)$ is

a) x = 0 b) y = 0 c) x = 1 d) x = – 1

12. The value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$ is

a) 0 b) e^u c) 1 d) 2

13. Volume of solid obtained by revolving the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major and minor axes are in the ratio

a) $b^2 : a^2$ b) $a^2 : b^2$ c) a : b d) b : a

14. The value of $\int_0^{\pi} \frac{\sin^2 x}{1 + \cos x} dx$ is

a) $\frac{\pi}{2}$ b) 0 c) $\frac{\pi}{4}$ d) π

15. The particular integral of $(3D^2 + 4D + 1)\, y = 3e^{-x/3}$ is

a) $\frac{2}{3}e^{-x/3}$ b) $\frac{x^2}{2}e^{-x/3}$ c) $\frac{3}{2}xe^{-x/3}$ d) $e^{-x/3}$

16. If $f'(x) = \sqrt{x}$ and f(1) = 2 then, f(x) is

a) $\frac{-2}{3}(x\sqrt{x} + 2)$ b) $\frac{3}{2}(x\sqrt{x} + 2)$ c) $\frac{2}{3}(x\sqrt{x} + 2)$ d) $\frac{2}{3}x(\sqrt{x} + 2)$

17. Which of the following statements is not correct

a) Matrix addition is a binary operation on the set of m × n matrices

b) Matrix addition is a binary operation on the set of n × n singular matrices

c) Matrix multiplication is a binary operation on the set of singular matrices

d) Matrix multiplication is a binary operation on the set of non singular matrices

18. Which of the following is a group

a) (N, +) b) (E, •) c) (R, •) d) (Q, +)

19. If a random variable X follows poisson distribution such that $E(X^2) = 30$ then the variance of the distribution is

a) 6 b) 5 c) 30 d) 25

20. A random vaiable X has the following probability mass function as follows

X	0	1	2
P (X = x)	$\frac{144}{169}$	$\frac{1}{169}$	$\frac{24}{169}$

Then the value of F(1) is

a) 1 b) $\frac{144}{169}$ c) $\frac{145}{169}$ d) $\frac{168}{169}$

Part - II

Answer any seven questions. Question no. 30 is compulsory. **7 × 2 = 14**

21. Show that the lines $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ are not coplanar.

22. Find the multiplicative inverse of the complex number 1 + i.

23. Define Director Circle.

24. Evaluate : $\lim_{x\to\infty} \frac{\sin 1/x}{1/x}$

25. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 1%.

26. Find the volume of the solid that results when the region enclosed by the given curve $y = x^3$, $x = 0$, $y = 1$ is revolved about y axis.

27. Solve : $xdy - ydx = x^2dx$

28. Prove that (N, *) where * is defined by $a * b = a^b$ is not a semigroup.

29. Prove that the total probability in a poisson distribution is one.

30. Solve the matrix equation XA = B where $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

PART - III

Answer any seven questions. Question no. 40 is compulsory. **(7 × 3 = 21)**

31. Find the rank of the matrix $\begin{pmatrix} 1 & -2 & 3 & 4 \\ 3 & 0 & 1 & 2 \\ -2 & 4 & -1 & -3 \\ 1 & 2 & 7 & 6 \end{pmatrix}$

32. What is the unit vector that is normal to the plane containing $2\vec{i} + \vec{j} + \vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$

33. Prove that the product of perpendiculars from any point on the hyperbola to its asymptotes is constant and the value is $\frac{a^2b^2}{a^2 + b^2}$.

34. Prove that $e^x > 1 + x$ for all $x > 0$.

35. Discuss the curve $3ay^2 = x (x - a)^2$ for (i) existence (ii) symmetry (iii) asymptote

36. Evaluate the integral $\int_1^5 |x - 3|\, dx$.

37. The normal lines to a given curve at each point (x, y) on the curve pass through the point (2, 0). Formualte the Differential equation representing the problem.

38. Write down the negation of the statement $(p \to q) \vee (q \to p)$.

39. A continuous random variable X has the p.d.f defined by $f(x) = \begin{cases} \frac{1}{2}\sin x & 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$. Find the mean of the distribution.

40. Find all the values of $(-i)^{1/3}$.

PART - IV

Answer all the questions. **(7 × 5 = 35)**

41. a) Determine the values of K such that the system of linear equations $x - 2y = 1$, $x - y + Kz = -2$, $Ky + 4z = 6$ has (i) a unique solution (ii) infinite number of solutions (ii) no solution

(or)

b) Prove that the perpendicular bisectors of sides of a triangle are concurrent by vector method.

42. a) If $\arg\left(\frac{Z-1}{Z+1}\right) = \frac{\pi}{2}$ then prove that $|Z| = 1$.

(or)

b) Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 y}{\partial y \partial x}$ for the function $u = \sin xy$.

43. a) A Kho-Kho player in a practice session while running realises that the sum of the distances from the two Kho-Kho poles from him is always 8m. Find the equation of the path traced by him if the distance between the poles is 6m.

(or)

b) Solve the differential equation :

$$\left(D^2+5\right)y=\left(\sqrt{\frac{1+\sin x}{1-\sin x}}+\sqrt{\frac{1-\sin x}{1+\sin x}}\right)\cos^2 x$$

44. a) Find the cartesian equation of the plane passing through the point (–1, 3, 2) and perpendicular to the planes x + 2y + 2z = 5, 3x + y + 2z = 8. Also find the intercepts with the three coordinate axes made by the plane.

(or)

b) Find the condition for the curves ax2 + by2 = 1, a1x2 + b1y2 = 1 such that the tangent lines at their point of intersection are perpendicular.

45. a) Find the surface area of the solid generated by revolving the arc of the parabola y2 = 4ax bounded by its latus rectum about x axis.

(or)

b) Find the equation of the hyperbola whose foci are $(\pm\sqrt{10},0)$ and passing through (3, 2).

46. a) Show that the 6th roots of unity form an abelian group with usual multiplication.

(or)

b) The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement within 12 months.

47. a) Find the point on the parabola $x + y^2 = 0$ that is closest to the point (0, –3) .

(or)

b) A radioactive substance disintegrates at a rate proportional to its mass. When its mass is 10mgm, the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10mgm to 5mgm ($\log_e 2 = 0.6931$).